PERTAINING TO THE THEORY OF DIFFUSION TRANSPORT IN MATERIALS WITH TRAPS

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The relaxation of the processes of absorption and release of diffusing atoms from structural defects in a solid material, dependent on the duration of the laxation times, leads to a hyperbolic or elliptical equation to describe the diffusion process.

The internal heterogeneity and structural imperfection of metals and alloys exert a marked influence on the behavior and, in particular, on the diffusion transport of impurities dissolved within these [1]. These defects are capable of absorbing and retaining within the flow (for a specific period of time) the atoms of the diffusing impurity, thus simultaneously preventing their migration and affecting both the velocity and other characteristics of the diffusion process. In addition to macroscopic defects (extraneous inclusions, elements of the new phase, micropores, grain boundaries, etc.), real crystals also usually contain microscopic "zero dimensional" defect (vacancies of various kinds, impurity atoms, broadened interstices and interstices in general through which no migration takes place, etc.), capable of absorbing only a single atom. Although the types of such spot defects are extremely varied [2], they can virtually all be represented as low-energy traps (see Fig. 1), whose characteristic dimensions are on the order of the interatomic distances within a crystal, with the binding energy considerable greater than that for impurity atoms in their normal positions. The dimensions of the macroscopic traps may exceed the interatomic distances by several orders of magnitude. In this case, the situation depicted in Fig. 1 characterizes the transition of atoms through a potential barrier at the surfaces of the traps.

The phenomenal logical theory of diffusion in materials with spot traps, based on the modeling of capture and release processes in the case of diffusing atoms resulting from second-order reactions with nonvarying reaction-rate constants, was developed in [3-7] and, subsequently utilized rather extensively (see, for example [8-10]). This theory is based on the equations

$$\begin{gathered}\partial c/\partial t = D\Delta c - k_1(m_* - m)\,c + k_2(c_* - c)\,m,\\ \partial m/\partial t = k_1(m_* - m)\,c - k_2(c_* - c)\,m.\end{gathered} \tag{1}$$

In situations in which the fraction of the normal occupied positions and traps is small in comparison to unity (i.e., $c \ll c_*$, $m \ll m_*$), system (1) is easily linearized, assuming D to be independent of c, i.e.,

$$\partial c/\partial t = D\Delta c - kc + \lambda m, \quad \partial m/\partial t = kc - \lambda m, \tag{2}$$

where $k = m_* k_1$, $\lambda = c_* k_2$. When we assume that the nonsteadiness is weak, from (2) we can obtain [1, 3-7]:

$$\frac{\partial (c+m)}{\partial t} = D\Delta c \approx \frac{D}{1 + k/\lambda}\,\Delta (c+m), \tag{3}$$

which approximately describes the reduction in the coefficient of diffusion, if we define the diffusion flux on the basis of the total concentration gradient rather than on the basis of the concentration gradient applicable to the atoms in normal positions, provided that we take into consideration those atoms retained within the traps.

The quantities k_1 and k_2 describe, respectively, the processes of atomic absorption and release by occupied spot traps, each of which, i.e., the processes, consists of two successive stages. The first process involves the diffusion migration of an atom in the immediate vicinity of the trap and the actual capture of that atom by the trap (the overcoming by the

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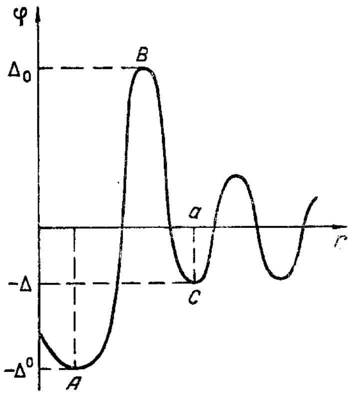

Fig. 1

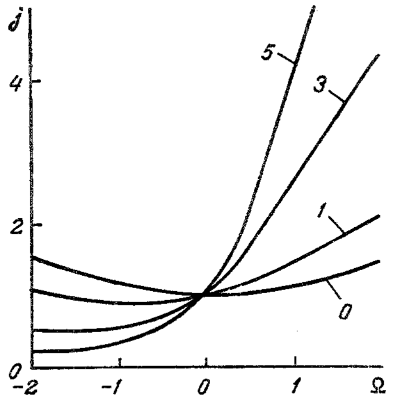

Fig. 2

Fig. 1. Energy circuit of trap.

Fig. 2. Relative flow in the pressure oscillation method as a function of the dimensionless frequency for various H (numerals identify the curves)

atom of the potential barrier $U = \Delta + \Delta_0$). The second process involves the release of the atom from the trap (the atom's overcoming of the barrier $U^\circ = \Delta^\circ + \Delta_0$) and its diffusion displacement through a distance considerably greater than a .

However, the diffusion stages of the two processes are extended both in space and in time. Therefore, their rates at a specified point at some instant of time must depend on the concentrations c and m not only at that point, but in its vicinity at all previous instants of time. Moreover, the stages associated with the overcoming of the potential barriers by the atoms represent, generally speaking, relaxation-type processes. Equations (2) with constant coefficients may therefore be valid only for steady-state diffusion processes, or states very close to these, while being physically inadequate for significantly nonsteady diffusion processes. As follows from results of an analysis of the mass and heat transfer in heterogeneous dispersed systems [11, 12], these coefficients must not be treated as constants, but rather as integral differential operators over time, whose form actually determines the nature of the transport process.

In an approximation leading to (2), these operators must be linear. Therefore, it is natural to apply the Fourier transform (2) insofar as this pertains to time, transforming these operators into functions of the transformation (frequency) parameter ω [11, 12], i.e., to obtain

$$i\omega c = D\Delta c - kc + \lambda m, \quad i\omega m = kc - \lambda m. \tag{4}$$

Instead of (4), we might also take a look at the unique equation

$$\Delta c - s^2 c = 0, \quad s^2 = \frac{i\omega}{D}\left(1 + \frac{k}{\lambda + i\omega}\right) \tag{5}$$

(the notations for the Fourier transforms remain the same as for the originals).

In the following we will assume that the concentration m_* for the spot defects is small in comparison with the concentration c_* for the normal diffusion positions. Then, first of all, it is possible to neglect the difference in the effective coefficient of diffusion under steady-state conditions from the coefficient of diffusion in the matrix (i.e., in the material which contains no defects). Secondly, Eq. (5) can be used to describe the diffusion of atoms near each defect in accordance with the classical Smolukhovskii method [13, 14], assuming the defect to be situated in the virtually unbounded matrix.

Let us now take a look at the three-dimensional and surface macroscopic traps. For the sake of determinacy, we will simulate these, respectively, by a sphere of radius a or by a spherical layer of the same radius, with thickness δ, containing an impurity uniformly distributed over the sphere or layer with the concentration c°. The quantity m, representing in this case the ratio of the total number of atoms in all of the traps in a small physical volume of material to the magnitude of this volume, is equal to

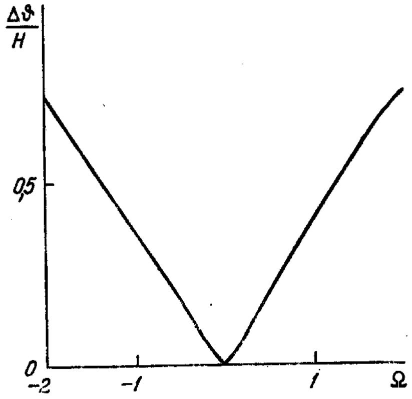

Fig. 3

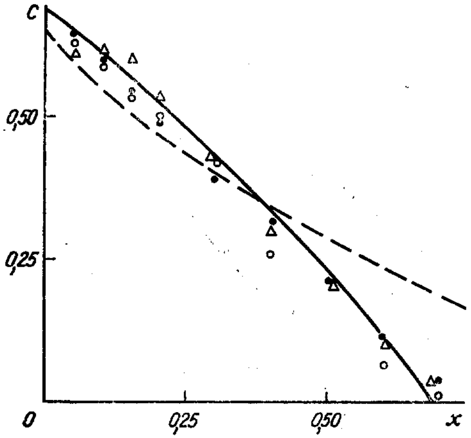

Fig. 4

Fig. 3. Flow phase in pressure oscillation method as a function of frequency.

Fig. 4. Carbon concentration relative units (distribution through the thickness of the surface steel layer) the points indicate results from various experiments under identical conditions (and theoretical curves taken from hyperbolic and parabolic equations) the solid and dashed curves, respectively (in intervals of 4.67 h after the onset of saturation. x, mm.

$$m = 4\pi a^2 \delta n c^{\circ}, \tag{6}$$

with $\delta = a/3$ for the three-dimensional traps and δ for those surface traps independent of

As follows from theory [15, 16], atomic diffusion in the vicinity of each of the traps can also be described in this case, on the average, by means of equations such as (4) and (5). In this case, the presence of other traps affects this process in two ways. On the one hand, the effects of contriction lead to a dependence in these equations of D on ρ. On the other hand, the existence of sources and sinks of diffusing atoms at the trap surfaces results in dispersion and an average translational diffusion flux of atoms in the hetereogeneous medium, and an average three-dimensional flow between phases. As a result D, k and λ in (4) and (5) are also dependent on ω.

The constriction-based diffusion-coefficient correction factors exhibit an order ρD, while the time scales characterizing the dispersion effects are of order $\rho a^2/D$. Even when $\rho \sim 1$ these times are considerably shorter than the characteristic times of the diffusion processes in metals and other solid materials. Indeed, as a suitable estimate of the size of the macroscopic defects we can take $a \sim 10^{-7}$-10^{-6} m [1]. Then $a^2/D \sim 1$ only when $D \sim 10^{-14}$-10^{-12} m^2/sec, which corresponds to diffusion at relatively low temperatures. In the temperature range 10^2-10^3 °C, in which the chemicothermal treatment of the metals is usually conducted, $D \sim 10^{-11}$-10^{-9} m^2/sec, i.e. the time a^2/D, and even $\rho a^2/D$, amounts to only small fractions of a second, which is considerably less than the tens of minutes or even hours needed for the chemicothermal treatment.

Therefore, for the sake of simplicity, in the following we will totally neglect the indicated dispersion effects. This will make it possible to significantly simplify the problem since the analysis of these effects is complex and cumbersome [15, 16]. Moreover, we will assume that $\rho \ll 1$ (this is equivalent to the assumption that $m_* \ll c_*$ for spot traps) and we will neglect the possible change in the dimensions of the traps in the diffusion process. This will offer us the possibility of leaving out of our consideration the effect of constriction on the value of the effective diffusion factor D, assuming it to be coincident with the diffusion factor for the impurity in the matrix.

Thus, under the adopted assumptions, the only reason for the appearance of the dispersion effect [the dependence on ω of k and λ in (4) and s^2 in (5)] can involve only the influence exerted by the relaxation processes on the formation of nonsteady processes by which the atoms overcome the potential barriers in the case of spot trap collectors and for the case in which these atoms pass through the thin layer of interphase boundary in the case of macroscopic collector traps. In the first case, relaxation is associated with the fact that in the nonsteady process the conditions of quasisteadiness for the Kramers model of transition through the potential barrier are, generally speaking, not fulfilled. In the second case, relaxation can be ascribed to the finiteness of the rate at which the composition of the indicated layer is formed, and this corresponds to the varying concentrations within the volume of the phases separated by this layer. Since the various types of collectors are extremely diverse, we will make use here of the simplest relaxation phenomenological relationship

$$\tau_j \partial q_j/\partial t + q_j = Q_j \tag{7}$$

for nonsteady flows q_j, which in the case of j = 1, 2 respectively characterize the capture and release of atoms by the collectors without making provision for atomic diffusion. The quantities Q_j represent the steady-state values of the flows corresponding to the given instantaneous conditions, while τ_j corresponds to the corresponding empirical relaxation times. Depending on the type of trap, for purposes of calculating these quantities, we must either treat the dynamic problems dealing with transition through the barriers in terms of the methods of physical kinetics [13, 14], or examine the dynamics of change of state in the interphase separation layer by making provision for the processes of sorption and descrption in each of the phases, as well as the processes of dissociation and recombination and the possible chemical conversions within the layer itself [17, 18].

Let us introduce a coordinate system whose origin is at the center of one of the collector traps and we will write the Fourier transform for concentration unperturbed by this collector in the form of Taylor expansion

$$c(\omega, \mathbf{r}) = c + \mathbf{E}\mathbf{r} + \mathbf{r}\mathbf{M}\mathbf{r} + \ldots, \quad \mathrm{Sp}\,\mathbf{M} = s^2c/2, \tag{8}$$

in which c, **E** , and **M** have been determined at the point r = 0, while the relationship for the trace of the tensor **M** follows directly for (5). In actual fact, Eq. (8) in the vicinity of the trap ($r \sim a$) represents an expansion over powers of the small a/L, where L is the linear dimension of the unperturbed field. The concentration $c^\circ = c^\circ(\omega)$ of the impurity within the trap depends, generally speaking, on its location within the heterogeneous medium; in this regard, it is no different from the quantity $c = c(\omega)$, introduced in (8).

The presence of a collector leads to the perturbation $c'(\omega, \mathbf{r})$ of the quantity $c(\omega)$, which must also satisfy Eq. (5). Taking Eq. (7) and (8) into consideration, we arrive at the following problem for the field $c'(\omega, \mathbf{r})$:

$$\Delta c' - s^2c' = 0, \quad r > a; \quad \lim_{r\to\infty} c' = 0; \quad D(\mathbf{r}/r)\,\nabla(c + c')_{r=a} = \Lambda_1(c + c')_{r=a} - \Lambda_2 c^\circ, \tag{9}$$

into which we have introduced the complex atomic capture and absorption reaction rates for the traps, namely,

$$\Lambda_j = \sigma_j(1 + i\omega\tau_j)^{-1}, \quad j = 1,\ 2, \tag{10}$$

where the velocity constants σ_j are in obvious fashion related to the empirical kinetic coefficients introduced within the framework of the phenomenological theory of diffusion [19]. In the following, these constants, as well as other introduced collector characteristics, are assumed to be known.

In view of the linearity of the equations, it would be possible to examine the processes of trap absorption and emission of atoms on an individual basis, as this is usually done in the theory of colloid coagulation (see, for example, [20, 21]).

It is natural to look for the solution of problem (9) in the standard form of an expansion over spherical functions whose form is dictated by an analogous expansion of series (8), such as which occurs under the boundary condition. The ultimate goal of this solution is the calculation of the impurity flow to the collector trap. It is obviously only the first spherical symmetrical term of this expansion that contributes to this flow. After we have written the expression exclusively for this term in explicit form, we obtain

$$c'(\omega, \mathbf{r}) = -[\Lambda_1 a + D(1 + z)]^{-1}\{\Lambda_1 a(1 + z^2/6) - Dz^2/3]\,c -$$
$$-\Lambda_2 ac^\circ\}\,(a/r)^{1/2}\exp[-s(r - a)] + \ldots$$

From this we have representation of the sort flow to the traps. If the collector traps are macroscopic, in the calculation we have the following per unit volume

$$J = nD \int\limits_{r=a} \frac{\mathbf{r}}{r} \nabla (c + c')\, dr = 4\pi a D [\Lambda_1 a + D(1+z)]^{-1} [\Lambda_1 a (1 + z + z^2/2 + z^3/6)\, c - \Lambda_2 a c^{\circ}], \quad z = sa, \quad n = 3\rho/4\pi a^3. \tag{11}$$

By definition, this quantity must be equal to $kc - \lambda m$, from which with consideration of (6) we have

$$k = 4\pi a n D \left(1 + z + \frac{z^2}{2} + \frac{z^3}{3}\right) \frac{\Lambda_1 a}{\Lambda_1 a + D(1+z)}, \tag{12}$$

$$\lambda = \frac{D}{a\delta}(1+z) \frac{\Lambda_2 a}{\Lambda_1 a + D(1+z)}.$$

On the other hand, from the determination of s^2 in (5) we obtain

$$k = (\lambda + i\omega)\left(z^2 - \frac{i\omega a^2}{D}\right) \frac{D}{i\omega a^2}, \quad z = sa. \tag{13}$$

Using the expressions for k and λ from (12) here, we subsequently obtain an algebraic equation for z, which we will write out as follows, introducing

$$\varepsilon = \frac{\omega a^2}{D}, \quad \xi = \frac{a}{\delta} \frac{\sigma_2}{\sigma_1}, \quad \eta = \frac{D}{a\sigma_1}, \quad \theta_j = \frac{\tau_j D}{a^2}, \tag{14}$$

in dimensionless form

$$i\varepsilon\rho (1 + i\varepsilon\theta_2)(1 + z + z^2/2 + z^3/6) = (z^2 - i\varepsilon)\{\xi(1 + i\varepsilon\theta_1)(1+z) + i\varepsilon(1 + i\varepsilon\theta_2)[1 + \eta(1 + i\varepsilon\theta_1)(1+z)]\}. \tag{15}$$

In the general case, from (15) we can derive a system of two nonlinear algebraic equations for the real and imaginary parts of z, in whose solution it is necessary to resort to numerical methods. However, let us note that for processes not markedly different from the steady state, the dimensionless frequency must be small in comparison with unity. Therefore, it is natural to seek the solution of (15), using the series

$$z = \sqrt{i\varepsilon}\,(C_0 + C_1\sqrt{i\varepsilon} + C_2 i\varepsilon + \ldots), \tag{16}$$

$$z^2 = i\varepsilon\,[C_0^2 + 2C_0C_1\sqrt{i\varepsilon} + (C_1^2 + 2C_0C_2)\, i\varepsilon + \ldots].$$

Having substituted (16) into (15) and having equated the various individual powers of $i\varepsilon$ in the left- and right-hand parts, we derive a system of real algebraic equations for the coefficients C_j, which are easily solved consecutively. This defines the expansion coefficients z^2 in (16), as well as the corresponding expansion of the quantity $s^2 = (z/a)^2$. Using the latter in Eq. (5) and limiting ourselves to the accuracy of terms of order ε^2 inclusively, we derive the following equation:

$$i\omega(1 + i\omega T)\, c = D_* \Delta c. \tag{17}$$

Here we have introduced the coefficients

$$D_* = \frac{\xi}{\rho + \xi} D, \quad T = \frac{\rho}{\xi(\rho+\xi)} \left[\frac{\rho+\xi}{2} - 1 - \eta + \xi(\theta_2 - \theta_1)\right] \frac{a^2}{D}. \tag{18}$$

If we examine the spot collectors, then in the calculation of the flow J we must take into consideration that $m_* - m \approx m_*$ empty collectors are capable of absorbing atoms per unit volume of heterogeneous material, while m occupied collectors can emit atoms. In this case, c° is formally equal to $3/4\pi a^3$ for the occupied collectors and to zero for the empty collector traps. The expressions for k and λ, as before, are given by formulas (12), if n is replaced by m_* in the latter and it is assumed that $\delta = a/3$, as is the case for macroscopic three-dimensional traps. Thus, Eq. (17) with the coefficients of (18) is approximately valid also for materials containing spot collectors.

Inverting the Fourier transform, from (17) we have

$$T\frac{\partial^2 c}{\partial t^2} + \frac{\partial c}{\partial t} = D_* \Delta c, \tag{19}$$

where the effective diffusion coefficient D_* differs, generally speaking, from D, while $|T|$ plays the role of the characteristic time for the diffusion process. If we take into consideration in the calculation of the expansion coefficients in (16) of those terms proportional to $(i\varepsilon)^b$ with $b > 2$, then in (19) we note the appearance of terms with time derivatives higher than the second.

We can see from formulas for D_* in (18) that the fact that the diffusing impurity is exchanged between the matrix and the traps leads to a reduction in the effective observed diffusion factor relative to D, and this is all the more pronounced, the smaller ξ is in comparison with ρ. From the determination of ξ in (14) it is clear that this effect is more characteristic for three-dimensional (including spot) collector traps than it is for surface collector traps (for the latter we usually have $\delta \ll a$), and it occurs in the event that the atoms are absorbed considerably more easily by the traps than are emitted from the traps (i.e., $\sigma_1 \gg \sigma_2$). We should stress that this effect is due entirely to objective physical factors, and not associated exclusively with the overdetermination of the diffusion flow [as in the derivation of (3)]. This was noted earlier in [12, 16] for media in which the inclusions absorbed the impurites.

With ρ negligibly small in comparison with unity (this must be the case, since it was conditional earlier to make no provision for dispersion effects such as those dealt with in [11, 12, 16]), and for T within the limits of adopted accuracy we can write the formula

$$T = \left[-\frac{1+\eta}{\xi} + \frac{\rho}{\rho+\xi} (\theta_2 - \theta_1) \right] \frac{a^2}{D}, \tag{20}$$

from which it follows that $|T|$ also increases with a reduction in ξ. If the relaxation of the atom transition processes from the matrix to the collector and back is not significant $[(1 + \eta)/\xi \gg \theta_2 - \theta_1]$, Eq. (19) which describes the nonsteady diffusion process belongs to the elliptical type. In this case, the time $|T|$ for large ξ is proportional to $\eta/\xi \sim (\delta/a)\times (D/a\sigma_2)$. Consequently, it increases, first of all, with transition from surface to three-dimensional traps and, secondly, it increases when the rate constant for the emission of atoms from the traps diminishes relative to the diffusion rate D/a.

In the opposite extreme case $(1 + \eta/\xi \ll \theta_2 - \theta_1)$, Eq. (19) is hyperbolic with $\theta_2 > \theta_1$ $(T > 0)$ and elliptical with $\theta_2 < \theta_1$. From the physical point of view, the transition to the hyperbolic telegraph mass-transfer equation and the subsequent reduction in the rate of propagation for the concentrated waves described by these equations is equal to $(D_*/T)^{1/2}$, and when θ_2 increases relative to θ_1 quite understandable. Indeed, this last situation coresponds, roughly speaking, to an increase in the average stay time of the atoms in the traps and, thus, to a decrease in the effective velocity of the atoms executing random motions.

Let us note that the relaxation effects which we are studying here do not, unlike the dispersion effect in [11, 12, 16], lead to the appearance in (19) of integral terms describing the effect of its earlier history on the diffusion process. This corresponds to the situation in which the coefficient C_1 in (16) is identically equal to zero. However, the subsequent integrals appear in the equation derived by means of the Fourier transform from (5) when using the following s^2 expansion terms over the powers of $i\varepsilon$.

Equations (17) and (19) can be used directly in analyzing the propagation of concentration waves and various dispersion phenomena. For example, they are necessary for the correct interpretation of the experimental data obtained by means of the pressure, oscillation method [22], and for their evaluation on the basis of the diffusion coefficient, as well as possibly, other parameters. In the most widely used variant of this method, we investigate the flow of impurities through a plane plate on one of whose surfaces periodic conditions are specified. For the sake of determinacy, we will examine diffusion through the plate under the following boundary conditions:

$$c|_{x=0} = c_0 (1 + \beta \cos \omega t), \quad c|_{x=h} = 0, \quad 1 > \beta > 0. \tag{21}$$

From (17) and (21) we obtain

$$c/c_0 = 1 - x/h + \beta \mathrm{Re}\,[f \exp(i\omega t)], \quad d^2f/dx^2 = R^2 \exp(2i\psi)\, f, \; f|_{x=0} = 1,$$

$$f|_{x=h} = 0, \quad R = (\omega/D_*)^{1/2} (1 + T^2\omega^2)^{1/4}, \quad \psi = \frac{1}{2} \left[\pi - \mathrm{arctg} \left(\frac{1}{T\omega} \right) \right].$$

The solution of this problem is elementary, but cumbersome. Here we will write out only the approximate result for thick plates, when $hR \gg 1$:

$$f \approx \exp[-xR\exp(i\psi)] - \exp[-(2h-x)R\exp(i\psi)].$$

The impurity flow through the surface $x = h$ is equal to

$$J = -D_* \left.\frac{\partial c}{\partial x}\right|_{x=h} = \frac{c_0 D_*}{h} + \Delta J,$$

$$\Delta J \approx 2\beta c_0 D_* R \exp(-hR\cos\psi)\cos(\omega t - \vartheta), \quad \vartheta \approx hR\sin\psi - \psi \approx hR\sin\psi.$$

From this, we obtain

$$j = \frac{\langle \Delta J \rangle}{\langle (\Delta J)_0 \rangle} \approx \exp\left(h\frac{T\omega}{2}\sqrt{\frac{\omega}{2D_*}}\right), \quad \Delta\vartheta = \vartheta - \vartheta_0 \approx h\frac{|T|\omega}{2}\sqrt{\frac{\omega}{2D_*}},$$

where the angle brackets denote averaging over the period, while the zero subscripts indicates that the corresponding quantity is calculated at the limit $T_\omega \to 0$. It is easy to see that regardless of how small the frequency ω, for thick plates, we are dealing with a cumulative effect: given sufficiently large h the quantities $\langle\Delta J\rangle$ and ϑ may be arbitrarily different from $\langle\Delta J\rangle_0$ and ϑ_0. Therefore, it is clear that the presence of the term with the second derivative in (19) may be of decisive value for an adequate evaluation of the experimental result. If $|T|\omega \gtrsim 1$, we have

$$j \approx S\exp[-H(S\cos\Psi - \mathrm{sign}(\Omega)/\sqrt{2})], \quad \Delta\vartheta \approx H(S\sin\Psi - 1/\sqrt{2}),$$

$$S = (1+\Omega^2)^{1/4}, \quad \Psi = \frac{1}{2}\left[\pi - \mathrm{arctg}\,\frac{1}{\Omega}\right], \quad H = h\sqrt{\frac{\omega}{D_*}}, \quad \Omega = T\omega.$$

Figures 2 and 3 show j and $\Delta\vartheta/H$ as functions of Ω and H.

Equations (17), and then Eq. (19) also, were actually derived in the assumption that the quantity $|T|\omega$, where ω has the sense of the characteristic frequency of the diffusion process, is small in comparison with unity. Therefore, they must be valid, strictly speaking, only to describe those processes not significantly different from the steady state. However, if we understand D_* and T not necessarily to be the parameters determined in (18) and (20), but certain empirically chosen quantities, then these equations can, apparently, be used also for correlation of the experimental data derived in essentially nonsteady processes that are not described by the usual parabolic equation of diffusion [23].

As as example, Fig. 4 shows the results from one of the experiments dealing with the penetration of carbon into steel 40. We used cylindrical specimens 10 mm in diameter and 100 mm in length, the specimen were case hardened in a laboratory installation with a vacuum furnace in an endogas atmosphere to which methane has been added, the composition of the latter monitored by means of a chromatograph and a special electrochemical sensor. The conditions of the experiments at a temperature of 910° C were matched to the mass-transfer coefficient at the surface, equal to $0.525 \cdot 10^{-5}$ cm/sec, with a diffusion coefficient $1.78 \cdot 10^{-7}$ cm^2/sec, which was estimated on the basis of data for an unalloyed austenite, these data having been taken from [24]. We can see from Fig. 4 that the telegraph equation with a relaxation time of T = 2.85 h, chosen on the basis of the criterion best corresponding to the experimental points, considerably better describes the experimental distribution of carbon concentration in the surface layer of the specimen than does the parabolic equation with T = 0.

Further development of the proposed theory must be associated, apparently, with an analysis of relaxation processes for various types of collector traps.

NOTATION

a trap radius; c, c°, impurity atom concentrations in the matrix and within the collectors; c_*, concentration of positions accessible to atoms in the matrix; D, D_*, diffusion coefficients; h, plate thickness; J, flow in the direction of the traps per unit volume or flow through the plate; k, coefficient of trap absorption of the impurity; m, concentration of impurity contained within the traps, in a solid material; m_*, concentration of spot traps; n, numerical concentration of trap; q_j, Q_j, flows of impurities absorbed or emitted by the trap, and their steady-state values; s, parameter introduced in (5); T, characteristic time in (17) and (19); z, parameter determined in (11); δ, thickness of surface traps; ε, dimensionless frequency; η, ξ, parameters determined in (14); Λ_j, complex kinetic coefficients introduced in (10); λ, coefficient of trap emission of impurities; τ_j, θ_j, dimensional and dimensionless relaxation times for the processes of absorption and emission; ρ, volumetric concentration of collector traps; σ_j, absorption and emission rate constants; ψ, ϑ, phase displacements in the problem pertaining to the penetration of the impurity through the plate; ω, frequency.

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